



# Algorithms for AI and NLP (INF4820 — FSAs)

{ baa!, baaa!, baaaa!, ... }

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# Background: A Bit of Formal Language Theory

## Languages as Sets of Utterances

- What is a language? And how can one characterize it (precisely)?
  - simplifying assumption: language as a *set of strings* ('utterances');
- well-formed utterances are set members, ill-formed ones are not;
- provides no account of utterance-internal structure, e.g. 'subject';
- + mathematically very simple, hence computationally straightforward.



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## Regular Expressions

- Even simple languages (e.g. arithmetic expressions) can be infinite;
- to obtain a *finite description* of an infinite set → *regular expressions*.



# Brushing Up our Knowledge of Regular Expressions

`/[wW]oodchucks?/`

*woodchuck* — *Woodchuck* — *woodgrubs* — *woodchucks* — *wood*



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*ba!* — *baa!* — *baah!* — *baaaa!* — *baaaaaaaaaa!*



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*ba!* — *baa!* — *baah!* — *baaaa!* — *baaaaaaaaaa!*

*aa* — *aaa* — *aaaa* — *aaaaaa* — *aaaaaaaa* — *aaaaaaaaa* — ...



# Pattern Matching: Finite-State Automata

`/baa+!/`

*ba! — baa! — baah! — baaaa! — baaaaaaaaa!*



# Pattern Matching: Finite-State Automata

/baa+!/  
A rounded rectangular box containing the regular expression /baa+!/.

*ba!* — *baa!* — *baah!* — *baaaa!* — *baaaaaaaaaa!*

## Recognizing Regular Languages

- Finite-State Automata (FSAs) are *very restricted* Turing machines;
  - states and transitions: read one symbol at a time from input tape;
- *accept* utterance when no more input, in a ‘final’ state; else *reject*.





# Tracing the Recognition of a Simple Input

*/baa+!/*

*ba! — baa! — baah! — baaaa! — baaaaaaaaa!*

**Input Tape**

0	1	2	3	4
<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>!</i>



# A Rather More Complex Example

$/(aa)^+|(aaa)^+/$

*aa — aaa — aaaa — aaaaaa — aaaaaaaa — aaaaaaaaaa — ...*



# A Rather More Complex Example

$/(aa)^+|(aaa)^+/$

$aa — aaa — aaaa — aaaaaa — aaaaaaaa — aaaaaaaaa — \dots$

- Non-Deterministic FSAs (NFSAs): multiple transitions per symbol;  
→ a *search space* of possible solutions: decisions no longer obvious.



# Quite Abstractly: Three Approaches to Search

## (Heuristic) Look-Ahead

- Peek at input tape one or more positions beyond the current symbol;
- try to work out (or ‘guess’) which branch to take for current symbol.

## Parallel Computation

- Assume unlimited computational resources, i.e. any number of cpus;
- copy FSA, remaining input, and current state → multiple branches.

## Backtracking (Or Back-Up)

- Keep track of possibilities (*choice points*) and remaining candidates;
- ‘leave a bread crumb’, go down one branch; eventually come back.



# NFSA Recognition (From Jurafsky & Martin, 2008)

```
1  procedure nd-recognize(tape , fsa)  $\equiv$ 
2    agenda  $\leftarrow$  {⟨0, 0⟩};
3    do
4      current  $\leftarrow$  pop(agenda);
5      state  $\leftarrow$  first(current);
6      index  $\leftarrow$  second(current);
7      if (index = length(tape) and state is final state) then
8        return accept;
9      fi
10     for(next in fsa.transitions[state, tape[index]]) do
11       agenda  $\leftarrow$  agenda  $\cup$  {⟨next, index + 1⟩}
12     od
13     if agenda is empty then return reject; fi
14   od
15 end
```

